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Influence of the Thermodynamic Parameters and Geometry on the Degree of Separation and Energy Consumption in a Gas Thermal Diffusion Column under Static Conditions

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Abstract

The influence of the thermodynamic parameters of a binary gas mixture and the geometry of the column on the degree of separation and energy consumption in a gas thermal diffusion column (TDC) was investigated under static conditions (no gas production). The theoretical results are illustrated with experimental data for a nitrogen-helium mixture in a cylindrical TDC, in the temperature interval 300 to 800 K, and pressures ranging from 24 kPa to 1.1 MPa.

1. INTRODUCTION

Separation of gas mixtures is an essential process in today's chemical technologies. Of special interest is the problem of separation of inert gas mixtures, which is necessarily connected with physical methods of separation. A certain number of effects can be used as the basis for such methods. Such an effect is thermal diffusion.

Thermal diffusion as a phenomenon was discovered in 1856 by Ludwig (*1*), although the prospects for its use in gas technology only became real after the invention of the thermal diffusion column (TDC) by Clusius and Dickel (*2*). In principle, the TDC showed the possibility of consequent enrichment of vertical gas flows, induced by the natural convection in gravitational field in TDC, resulting from transverse thermal diffusion effect—a mass transfer process typically used in gas separation techniques.

2. DEGREE OF SEPARATION IN THERMAL DIFFUSION COLUMN.

The degree of separation Q in an apparatus for gas separation is defined as

$$Q = x_{L1}/x_{L2} \cdot x_{H2}/x_{H1} = Q_L Q_H \tag{1}$$

where

$$Q_L = x_{L1}/x_{L2} \tag{2}$$

$$Q_H = x_{H2}/x_{H1} \tag{3}$$

$$x_{L1} = 1 - x_{H1} \tag{4}$$

$$x_{L2} = 1 - x_{H2} \tag{5}$$

The general theory of the thermal diffusion column was developed by Furry, Jones, and Onsager (*3*). According to this theory, the degree of separation in a thermal diffusion column under static conditions (without gas production), Q_0, is determined from

$$Q_0 = \exp(Hh/K) \tag{6}$$

or

$$\ln Q_0 = Hh/K \tag{7}$$

where H and K are transport coefficients, introduced by Furry, Jones, and Onsager (*3*), which take into acccount the influence of diffusion, hydrodynamic, and thermal processes on the formation of the concentration distribution along the height of the column, and h is the total height of the column.

The coefficients H and K depend on the physical characteristics of the gas mixture, the temperature at which the process is carried out, and the

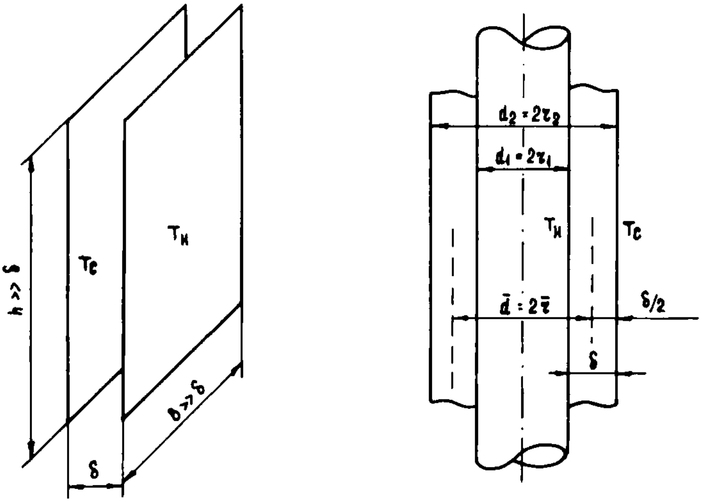


FIG. 1. Schematic drawing of a thermal diffusion column with flat-parallel and coaxial geometry.

geometry of the column. For flat or nearly flat geometry ($r_2/r_1 < 2.712$) (*4–6*) (see Fig. 1):

$$H = \varepsilon L = 2\varepsilon\rho^2 g\beta\Delta TB\delta^3/6!\eta \tag{8}$$

$$K = h_0L = K_c + K_d + K_p \tag{9}$$

$$K_c = h_cL = \rho^3g^2\beta^2\Delta T^2B\delta^7/9!\eta^2D \tag{10}$$

$$K_d = h_dL = \rho D\delta B \tag{11}$$

$$L = 2\rho g\beta\Delta TB\delta^3/6!\eta \tag{12}$$

$$h_c = \rho g\beta\Delta T\delta^4/1.4(6!\eta D) \tag{13}$$

$$h_d = 6!\eta D/2\rho g\beta\Delta T\delta^2 \tag{14}$$

$$h_0 = h_c + h_d + h_p \tag{15}$$

$$q = (T_H/T_c)^{\alpha}T \tag{16}$$

$$\varepsilon = (q - 1)/q = 1 - (T_c/T_H)^{\alpha_T} \tag{17}$$

$$\bar{T} = T_H T_c \ln (T_H/T_c)/(T_H - T_c) \tag{18}$$

$$\Delta T = T_H - T_c \tag{19}$$

Practically, in a thermal diffusion column only a half of ΔT is effectively used, so in a thermal diffusion column ΔT is the difference in the average temperatures T_{Hm} and T_{Cm} of up-going and down-going convection flows in the column

$$\Delta T = T_{Hm} - T_{Cm} \tag{20}$$

At a first approximation:

$$T_{Hm} = (T_H + \bar{T})/2 \tag{21}$$

$$T_{Cm} = (T_C + \bar{T})/2 \tag{22}$$

and therefore

$$\Delta T = (T_H - T_C)/2 \tag{23}$$

In the formal theory of mass-transfer processes, an effective transfer unit height (TUH) (*6*), a quantity with dimensions of length (h_0), is introduced. Physically, TUH can by interpreted as the effective height of the column at which an elementary part of the total concentration change for a chosen component takes place. By introducing TUH, the mass-transfer process can be described discretely as consisting of N stages, so that

$$N = h/h_0 \tag{24}$$

The coefficient K_p and the correction for TUH, namely h_p, do not follow from the theory of a thermal diffusion column. They describe the nonideal geometry and the variations of temperatures T_H and T_C resulting in the appearance of transverse diffusion flows. There are no reliable expressions for K_p and h_p. According to Vasaru et al. (*5*), the value of K_p can reach up to 20% of the value of $K_c + K_d$. This was confirmed experimentally by Sorensen et al. (*7*). Keeping in mind the above and Eqs. (9)–(11), the coefficient K can be written as (*8*)

$$K = k_p(K_c + K_d) = k_pL(K_c + K_d) \tag{25}$$

For an ideal column, $k_p = 1.0$. In practice, $k_p = 1.0$–1.2.

Substituting H and K from Eqs. (8)–(11) in Eq. (6), and by using Eqs. (13)–(15) and (25) for Q_0, one obtains the known expression (*6*):

$$\ln Q_0 = \varepsilon N \tag{26}$$

However, it was shown in Rozen's monograph (*6*) that Eq. (19) only holds for $\varepsilon \ll 1$ (mainly in isotope separation) and in general:

$$\ln Q_0 = \varepsilon N + \varepsilon \ln Q_L \tag{27}$$

or

$$\ln Q_H Q_L{}^{1-\varepsilon} = \varepsilon N \tag{28}$$

From Eqs. (6) and (28) it follows that

$$\ln Q_H Q_L{}^{1-\varepsilon} = Hh/K \tag{29}$$

which is a higher approximation of the Furry, Jones, and Onsager equation. From Eqs. (27)–(29) it is obvious that at $\varepsilon \ll 1$ (as in the case of isotope separation), Eq. (29) is reduced to Eq. (6), but for $\varepsilon > 0.1$, and for $QL \gg 1$ in the case of even lower values of ε, calculation of Q_0 from Eq. (6) can lead to large errors. As shown in Ref. 8, Eq. (6) is valid only when $x_0 \to 0$ and, respectively, $Q_L \to 1$ and $\ln Q_L \to 0$. In the opposite case $x_0 \to 1$, and, respectively, $Q_H \to 1$ and $Q_L \to Q_0$, it follows from eq. (6) that

$$\ln Q_0 = \varepsilon N/(1 - \varepsilon) = q\varepsilon N = (q - 1)N \tag{30}$$

In this case the error in calculating $\ln Q_0$ will be

$$\Delta = (q - 1)/q \cdot 100\% = \varepsilon \cdot 100\%$$

and for $\varepsilon > 0.1$, $\Delta > 10\%$, but in the determination of Q_0 (and not of $\ln Q_0$) the error is even greater.

Accordingly, neither Q_H nor Q_L can be calculated beforehand, and in Eq. (29) there are two unknowns.

Introducing the notation

$$m = \ln Q_H / \ln Q_L \tag{31}$$

from Eqs. (27)-(29) and (31), one obtains

$$\ln Q_0 = \bar{\varepsilon} N = \bar{H} h / K \tag{32}$$

and in this case

$$\bar{\varepsilon} = \varepsilon / [1 - \varepsilon/(m + 1)] \tag{33}$$

$$\bar{H} = H / [1 - \varepsilon/(m + 1)] \tag{34}$$

The determination of the ratio between Q_H and Q_L and thus the coefficient m is possible in principle if the initial concentration x_0, the geometry of the column, and the temperatures T_H and T_C are known, but the calculation is long and complicated. Therefore, ln Q_0 can be determined with an error of 2–3% except for $\varepsilon > 0.2$, and even then the error is less than 5% from:

$$0 < x_0 < 0.3;\ m \to \infty;\ \bar{\varepsilon} \approx \varepsilon;\ \ln Q_0 \approx \varepsilon N$$

$$0.3 < x_0 < 0.7;\ m \approx 1;\ \bar{\varepsilon} \approx \varepsilon/(1 - \varepsilon/2);\ \ln Q_0 \approx \varepsilon N/(1 - \varepsilon/2)$$

$$0.7 < x_0 < 1;\ m \approx 0;\ \bar{\varepsilon} \approx \varepsilon/(1 - \varepsilon) = q - 1;\ \ln Q_0 \approx (q - 1)N$$

2.1. Influence of the Geometry of Thermal Diffusion Column on the Degree of Separation

2.1.1. Influence of the Geometrical Height of the Column

It is obvious from Eq. (32) that for a height of the thermal diffusion column $h = 1.0$ m, the degree of separation will be

$$\ln Q_{01} = \bar{H}/K \tag{35}$$

where Q_{01} is the degree of separation for $h = 1.0$ m. From Eqs. (32) and (35):

$$\ln Q_0 = h \ln Q_{01} \tag{36}$$

or

$$Q_0 = Q_{01}{}^{h} \tag{36a}$$

i.e., the height of the thermal diffusion column very strongly affects the degree of separation. For example, doubling h leads to a Q_0 times increase of Q_0.

Because Eq. (36a) is of the type $y = x^a$ for $x > 1$ and $a > 0$, the function $Q_0(h)$ increases monotonously, and for the separation process there is no optimal, finite value for the height of the column for which Q_0 has a maximum ($Q_0 \rightarrow 1$ for $h \rightarrow 0$ and $Q_0 \rightarrow \infty$ for $h \rightarrow \infty$).

2.1.2. Influence of the Distance between the Hot and the Cold Wall of the Column: δ

Substituting the values of K and H from Eqs. (8)–(11) in Eq. (29) and using Eqs. (25), (32), and (34) for Q_0, one obtains

$$\ln Q_0 = \frac{(1.4)6!\bar{\varepsilon}h\eta D\rho g\beta\Delta T\delta^2}{k_p(\rho^2 g^2\beta^2\Delta T^2\delta^6 + 9!\eta^2 D^2)} \tag{37}$$

Assuming that k_p is independent of δ, it can be shown (4) that Eq. (37) has a maximum at

$$\delta = \delta_0 = [(9!/2)^{1/2}\eta D/\rho g\beta\Delta T]^{1/3} \tag{38}$$

Introducing a reduced value for δ:

$$\bar{\delta} = \delta/\delta_0 \tag{39}$$

and taking into account Eqs. (13), (14), (38), and (39), Eq. (37) can be written as

$$\ln Q_0 = 2\varepsilon h/k_p k_0\delta_0 + \varepsilon \ln Q_L = 2\bar{\varepsilon}h/k_p k_0\delta_0 \tag{40}$$

where

$$k_0 = (5/7)^{1/2}(\bar{\delta}^4 + 2/\bar{\delta}^2) \tag{41}$$

From Eqs. (32) and (40) it immediately follows that

$$h_0 = 0.5k_p k_0 \delta_0 \tag{42}$$

Using Expressions (38) and (39), Eqs. (8)-(14) can be written as

$$H = (7/5)^{1/2}\varepsilon\rho DB\bar{\delta}^3 \tag{43}$$

$$K = k_p(K_c + K_d) = 0.5(7/5)^{1/2}k_p k_0 \delta_0 \bar{\delta}^3 \rho DB \tag{44}$$

$$K_c = 0.5\rho DB\delta_0\bar{\delta}^7 \tag{45}$$

$$K_d = \rho DB\delta_0\bar{\delta} \tag{46}$$

$$L = (7/5)^{1/2}\rho DB\bar{\delta}^3 \tag{47}$$

$$h_c = (5/7)^{1/2}\delta_0\bar{\delta}^4 \tag{48}$$

$$h_d = (5/7)^{1/2}\delta_0/\bar{\delta}^2 \tag{49}$$

The variation of the coefficient k_0 as a function of the reduced parameter $\bar{\delta}$ is shown in Fig. 2. The strong dependence of k_0 on $\bar{\delta}$ is evident, as well as of Q_0 on $\bar{\delta}$, having in mind Eqs. (40) and (41). Therefore, in practice the coefficient $\bar{\delta}$ cannot be varied widely. As will be shown later, when the influence of the thermodynamical parameters of the gas mixture on the degree of separation, Q_0, is examined, the same limitation for δ follows from the thermodynamics of the column. It is therefore worthwhile to study the dependence of Q_0 on both δ and $\bar{\delta}$. For this reason the following considerations will be made for δ = constant as well as for $\bar{\delta}$ = constant.

2.2. Influence of the Thermodynamic Parameters of the Gas Mixture on the Degree of Separation in a Gas Thermal Diffusion Column

For gases and gaseous mixtures with an accuracy sufficient for qualitative evaluation, it can be assumed that

$$\rho = p/RT \tag{50}$$

$$\beta = 1/T \tag{51}$$

$$D = D_0(T/T_0)^{3/2}(p/p_0) \tag{52}$$

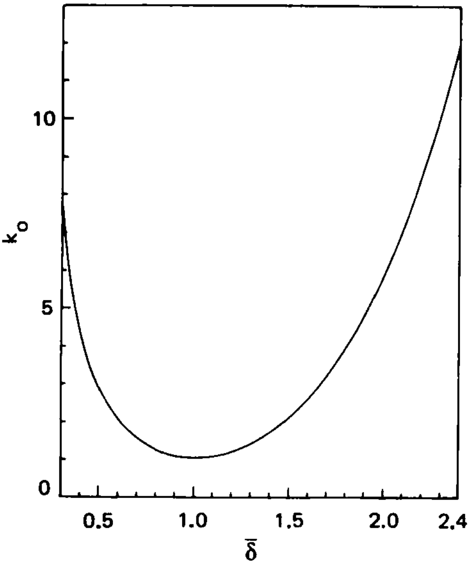


FIG. 2. Scale coefficient k_0 as a function of the relative distance between the hot and cold walls of the column $\bar{\delta}$.

$$\eta = \eta_0(T/T_0)^{3/2}(T_0 + C)/(T + C) \tag{53}$$

Substituting the values of ρ, β, D, and η from Eqs. (50)-(53) in Eqs. (37) and (38) one obtains, respectively,

$$\ln Q_0 = \frac{2\bar{\varepsilon} h_p^{2/3} \Delta T^{1/3}(\bar{T} + C)^{1/3}}{A_0 k_p k_0 \bar{T}^{5/3}} \tag{54}$$

$$\delta_0 = A_0 \bar{T}^{5/3}/p^{2/3} \Delta T^{1/3}(\bar{T} + C)^{1/3} \tag{55}$$

$$A_0 = [(9!/2)^{1/2} \eta_0 D_0 R(\bar{T} + C)/g T_0^5]^{1/3} \tag{56}$$

Obviously, A_0 is constant for any gas mixture.

2.2.1. Influence of the Initial Concentration (x_0) of the Light Component of the Gas Mixture

The physical parameters of a gas mixture, η_0, D_0, ρ, C, and α_T, depend on the proportion of the two gases in the mixture, i.e., on x_0. A_0, $\bar{\varepsilon}$, and $(T_0 + C)$

in Eqs. (54) and (55) also depend on the initial concentration, x_0, of the gas mixture to be separated. This is illustrated by the curves given in Fig. 3, which represent the functional dependence $\ln Q_0 = f(x_0)$ for the mixture He–N_2 at different pressures and average temperatures of the thermal diffusion process and at fixed δ and $\bar{\delta}$. Experiments were carried out on the apparatus described in Ref. *8*. The results for δ = constant are obtained directly in the experiments, while those for $\bar{\delta} = 1$ are calculated from these for δ = constant using Eq. (39).

The dependence of Q_0 on x_0 is stronger at higher pressures, where the difference between the minimum and maximum values reaches an order of magnitude (at 340 kPa and δ = constant).

Actually, in all practical problems x_0 is almost always a fixed parameter, more or less, and varies within relatively small limits (e.g., in a series of

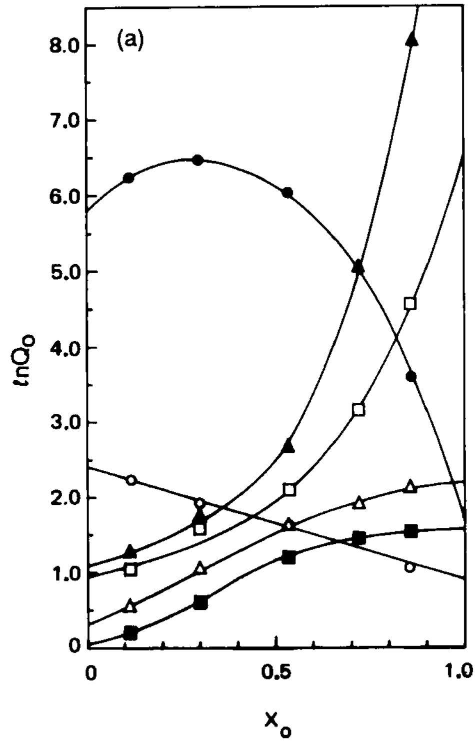


FIG. 3. Isobars of the degree of separation Q_0 as a function of the initial concentration of the light component of the He–N_2 mixture. x_0 at average temperature $\bar{T}$ = 398 K: (○) p = 24 kPa, (●) p = 51 kPa, (▲) p = 340 kPa, (□) p = 438 kPa, (△) p = 683 kPa, (■) p = 1076 kPa. (a) Dis-

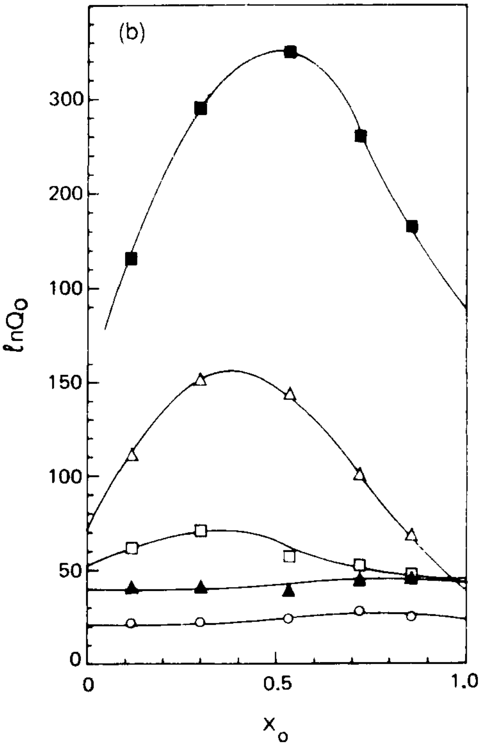
thermal diffusion columns) so that the thermal diffusion process can hardly be optimized by this parameter.

2.2.2. Influence of Pressure (p)

If the influence of pressure on α_T and k_p is neglected, it can be shown that the function ln Q_0 (Eq. 54) has a maximum at

$$p = (\delta^7 K_d/K_c)^{1/4} = [(9!)^{1/2}\eta D(p_0)/\rho(p_0) g \beta \Delta T \delta^3]^{1/4} \qquad (57)$$

(see also Ref. *4*).



tance between the hot and cold walls of the column, δ = 8 mm. (b) Relative distance between the hot and cold walls of the column, $\bar{\delta}$ = 1.

For mixtures of ideal gases the thermal diffusion factor α_T is independent of pressure, and in particular for isotope mixtures it can be written to a first approximation as (*3*)

$$\alpha_T = \frac{108}{118}\,\frac{M_H - M_L}{M_L + M_H} \tag{58}$$

For mixtures of ideal gases the thermal diffusion factor α_T is independent of pressure, and in particular for isotope mixtures it can be written to a first approximation as (*3*)

$$\ln Q_0 \sim \bar{\varepsilon}(p)p^{2/3}, \tag{59}$$

or, in relative units, for δ = constant:

$$\frac{\ln Q_0(p)}{\ln Q_0(p_o)} = \frac{\bar{\varepsilon}(p)k_0(p_0)}{\bar{\varepsilon}(p_0)k_0(p)}\,p^{2/3} \tag{60}$$

where p_0 = 100 kPa. For $\bar{\delta}$ = constant:

$$\frac{\ln Q(p)}{\ln Q_0(p_0)} = \frac{\bar{\varepsilon}(p)}{\bar{\varepsilon}(p_0)}\,p^{2/3} \tag{61}$$

The dependence of ln Q_0 on pressure at $\bar{\delta}$ = constant is shown in Figs. 4-6 for the He–N_2 mixture. The dependence of Q_0 on δ is well known (see, e.g., Refs. *4* and *5*). The experimental results of the authors, published in a previous work (*12*), are derived from an investigation of the thermal diffusion factor α_T for the He–N_2 mixture over large intervals of pressures, temperatures, and initial concentrations. As can be seen from the figures, the relation ln $Q_0 = f(p)$ has a small maximum at 30–100 kPa, a minimum at 100–500 kPa, and a high maximum at pressures of 0.8 to 2–3 MPa (when extrapolating to the higher limit). It is obvious that for the He–N_2 mixture the optimum pressure for maximum separation at given x_0 and $\bar{T}$ is in the range 0.8–3 MPa. It is interesting that the maximum values of α_T correspond to some other values of pressure (*12*).

Regarding the gas separation technology, an important fact is the existence of an optimum pressure which corresponds to maximum values of the degree of separation in a thermal diffusion column. This pressure is higher than that at which α_T, and so ε, reach their maximum values. Such a result, suggested by Eq. (59), is experimentally verified for the He–N_2 mixture, as seen from Figs. 4-6. There is no fundamental reason prevent-

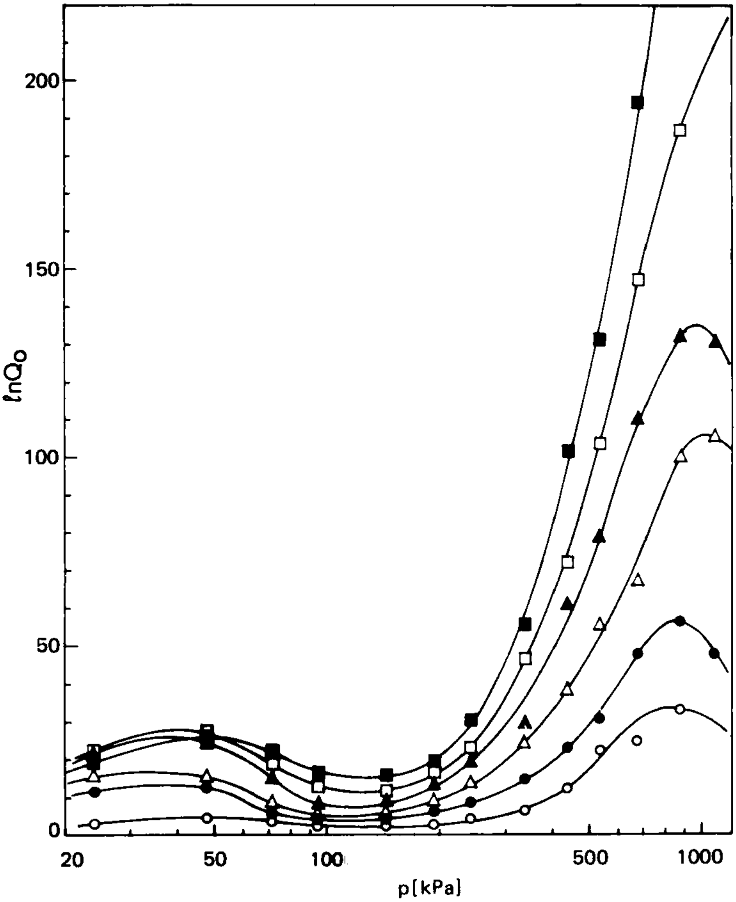


FIG. 4. Isotherms of the degree of separation Q_0 as a function of pressure p for the He–N_2 mixture with an initial concentration of the light component of $x_0 = 0.115$ at a relative distance between the hot and cold walls of the column of $\bar{\delta} = 1$ ($\bar{\delta} = \delta/\delta_0$): (○) $\bar{T}$ = 326 K, (●) $\bar{T}$ = 346 K, (△) $\bar{T}$ = 356 K, (▲) $\bar{T}$ = 398 K, (□) $\bar{T}$ = 426 K, (■) $\bar{T}$ = 454 K.

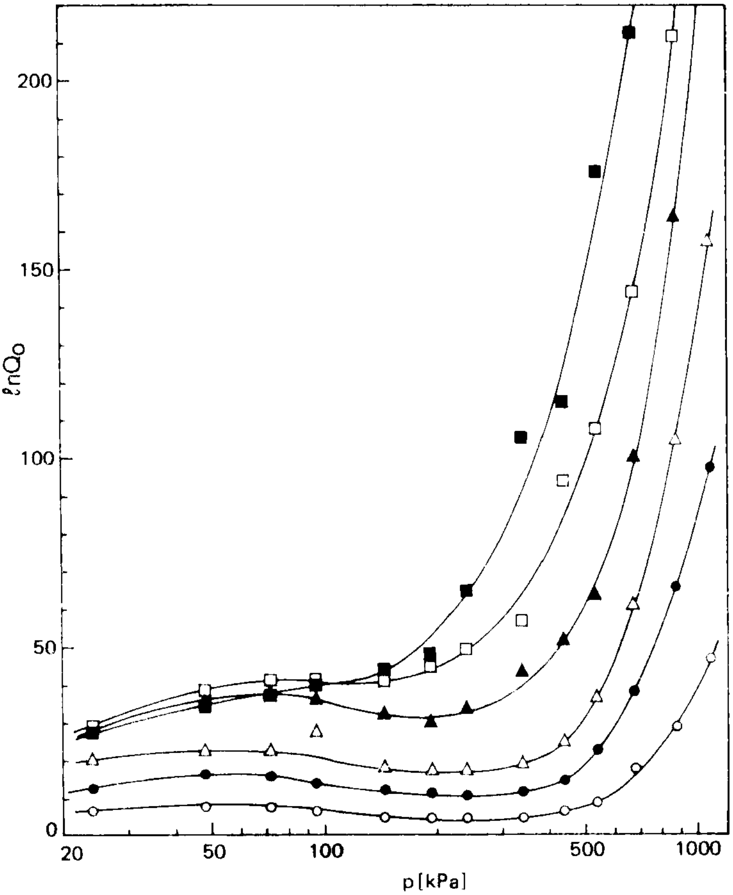


FIG. 5. Isotherms of the degree of separation Q_0 as a function of pressure p for the He–N$_2$ mixture with an initial concentration of the light component of $x_0 = 0.720$ at a relative distance between the hot and cold walls of the column of $\bar{\delta} = 1$ ($\bar{\delta} = \delta/\delta_0$): (○) $\bar{T}$ = 326 K, (●) $\bar{T}$ = 346 K, (△) $\bar{T}$ = 365 K, (▲) $\bar{T}$ = 398 K, (□) $\bar{T}$ = 426 K, (■) $\bar{T}$ = 454 K.

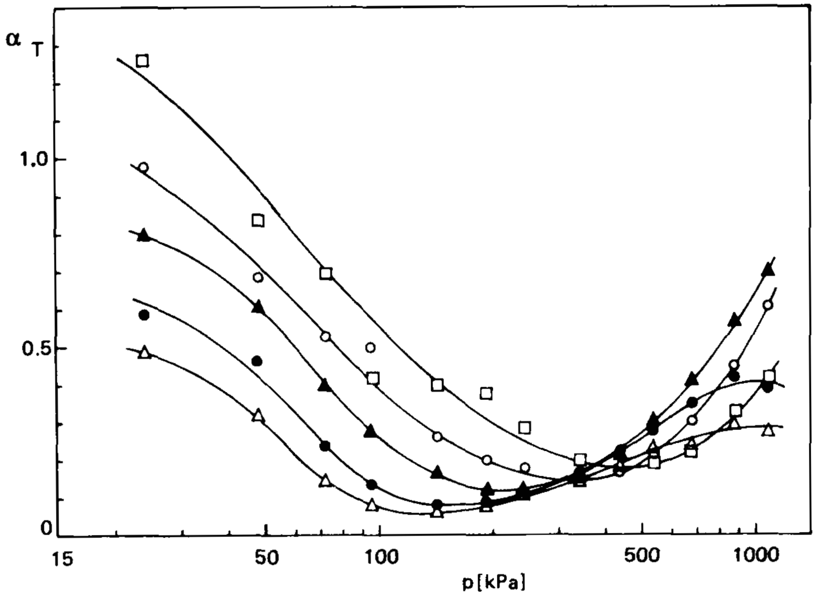


FIG. 6. Thermal diffusion factor α_T for a He–N_2 mixture as a function of pressure at average temperature $\bar{T}$ = 365 K: (△) x_0 = 0.115, (●) x_0 = 0.298, (▲) x_0 = 0.534, (○) x_0 = 0.720, (□) x_0 = 0.854.

ing the extension of these results to all binary gas mixtures by using Eqs. (56), (57), and (59). The practical limitation is only technical, so from Eq. (55) it follows that $\delta_0 \sim p^{-2/3}$ and for sufficiently high pressures one might obtain values for δ_0 that cannot be technically realized.

2.2.3. Influence of the temperatures T_H and T_C

To a first approximation from Eqs. (54) and (56) it follows that

$$\ln Q_0 \approx \bar{\varepsilon}(\bar{T})\Delta T^{1/3}(\bar{T} + C)^{1/3}/\bar{T}^{5/3} \qquad (62)$$

It is obvious that increasing ΔT at $\bar{T}$ = constant leads to an increasing Q_0 ($\ln Q_0 \approx \Delta T^{1/3}$), which is favorable for the separation process. On the other hand, experiments with the He–N_2 mixture showed (see Fig. 7), that $\ln Q_0/\Delta T^{1/3}$, which is a complicated function of $\bar{T}$ (see Eq. 62) at δ = constant, has a maximum in the range $\bar{T}$ = 400–600 K (depending on the initial con-

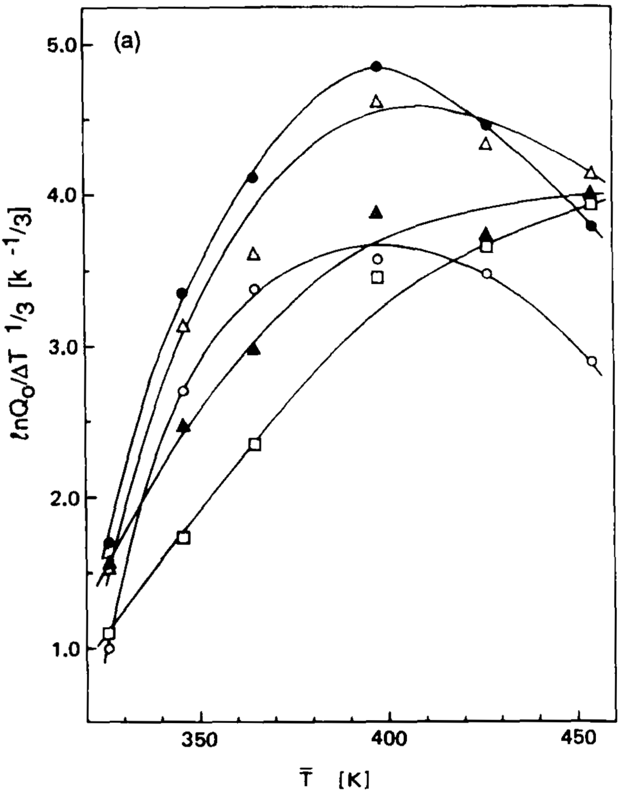
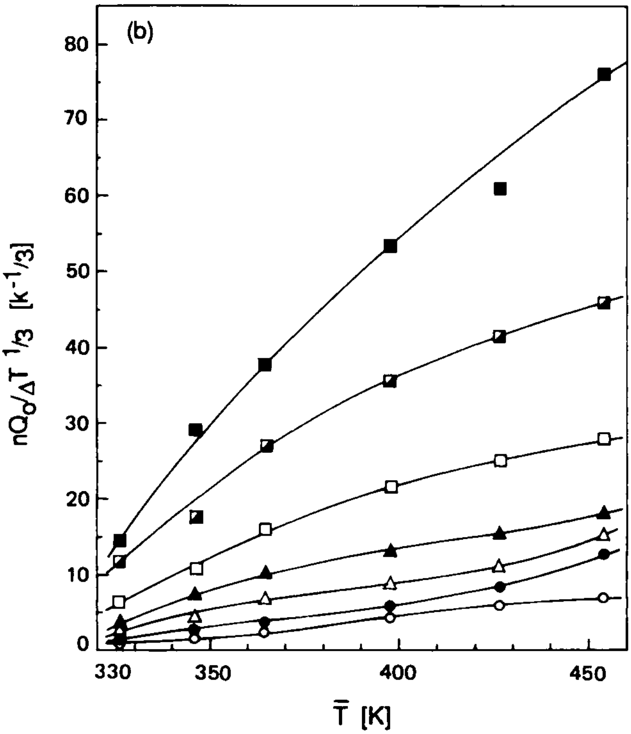


FIG. 7. Isobars of the degree of separation ln $Q_0/\Delta T^{1/3}$ as a function of the average temperature $\bar{T}$ for a He–N_2 mixture with an initial concentration of the light component of $x_0 = 0.534$ at an optimal distance between the hot and cold walls of $\delta = \delta_0\,(\bar{\delta} = 1)$. (a) (○) $p = 24$ kPa, (●)

centration and pressure). In the environment of optimum pressures, the region of maximum values of ln Q_0 as a function of $\bar{T}$ narrows and shifts to higher temperatures—between 500 and 600 K.

In practice, both from an energetic and technological point of view, it is most convenient to work at $T_C = 280$–300 K, because minimum energy consumption is needed to keep this temperature and it is technically easy to realize such conditions. Therefore it is interesting to determine Q_0 as a function of T_H at T_C = constant. This function is shown in Fig. 8 for the He–N_2 mixture. In this case $T_C = 287$ K and the optimum temperature of the hot wall T_H is the interval 600 to 1000 K, depending on pressure, and increasing when the pressure increases. So the optimum pressures for the



p = 48 kPa, (Δ)p = 72 kPa, (▲)p = 95 kPa, (□)p = 144 kPa. (b) (○)p = 242 kPa, (●)p = 340 kPa, (Δ) p = 438 kPa, (▲) p = 536 kPa, (□) p = 638 kPa, (□) p = 880 kPa, (■) p = 1076 kPa.

gas mixture under investigation lie above 800 kPa, and the corresponding optimum temperatures are higher than 800 K. In practice, this means that it is favorable to work at optimal values of T_H, which is technically feasible.

3. ENERGY CONSUMPTION OF THERMAL DIFFUSION COLUMN

The energy consumption of a thermal diffusion column means the amount of heat that the gas mixture carries from the hot to the cold wall of the column. This is calculated with the well-known formula

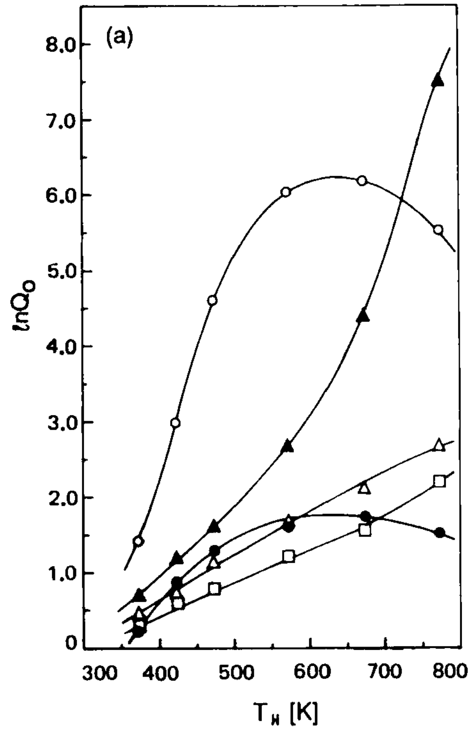


FIG. 8. Isobars of the degree of separation ln Q_0 as a function of the hot wall temperature T_H and the constant cold wall temperature T_C = 287 K for a He-N_2 mixture with an initial concentration of the light component of x_0 = 0.534. (●) p = 24 kPa, (○) p = 51 kPa, (▲) p = 340

$$E = \lambda(F/\delta)(T_H - T_C) \tag{63}$$

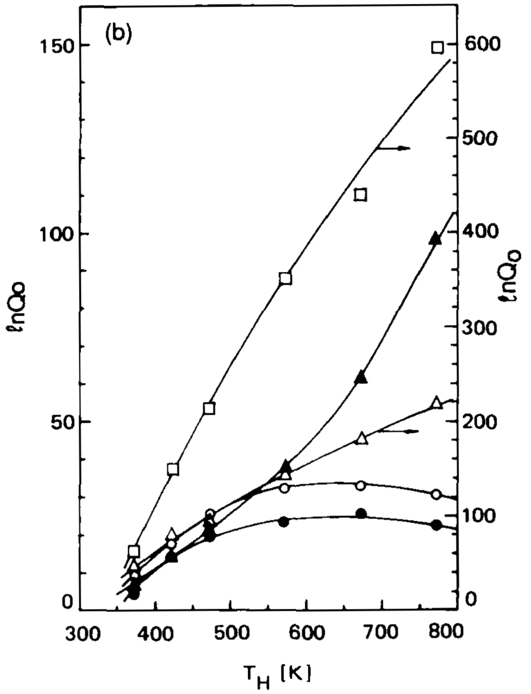
where

$$F = Bh \tag{64}$$

For cylindrical geometry:

$$B = 2\pi\bar{r} = \pi(r_1 + r_2) \tag{65}$$

Of course, the real energy consumption in a thermal diffusion column almost coincides with the one calculated by Eq. (63) only when T_C is equal



kPa, (△) p = 683 kPa, (□) p = 1076 kPa. (a) Distance between the hot and cold walls of the column: $\delta = 8$ mm. (b) Optimal distance between the hot and cold walls of the column: $\delta = \delta_0$ ($\bar{\delta} = \delta/\delta_0 = 1$).

to the temperature of the surroundings and the energy consumption for its maintenance is negligible compared to the total energy consumption.

For gases, one can assume with a sufficient accuracy that

$$\lambda = \lambda_0(T/T_0)^{3/2}(T_0 + C_1)/(T + C_1) \tag{66}$$

Using Eqs. (50)–(53) and (66), Eq. (63) can be written as

$$E = A_1 \frac{hp^{2/3}\Delta T^{4/3}(\bar{T} + C)^{1/3}}{\delta\bar{T}(\bar{T} + C_1)} \tag{67}$$

$$A_1 = \frac{\lambda_0 B(T_0 + C_1)}{[(9!/2)^{1/2}\eta_0 D_0(\bar{T}_0 + C)/g\bar{T}_0]^{1/3}} = \text{constant} \tag{68}$$

As was shown above, the case of interest for gas separation technology is when $\bar{\delta}$ = constant.

Assuming Q_0 = constant (i.e., a concrete practical problem) and that every change in p, $\bar{T}$, and ΔT is compensated for by changing the height of the column h so that Q_0 = constant, from Eq. (54) it follows that at $\bar{\delta}$ = constant:

$$\text{At } \bar{T} = \text{constant and } \Delta T = \text{constant:} \quad h \approx 1/\bar{\varepsilon}p^{2/3} \tag{69}$$

$$\text{At } p = \text{constant and } \bar{T} = \text{constant:} \quad h \approx 1/\Delta T^{4/3} \tag{70}$$

$$\text{At } p = \text{constant and } \Delta T = \text{constant:} \quad h \sim \bar{T}^{5/3}/\bar{\varepsilon}(\bar{T} + C) \tag{71}$$

By using Eqs. (69)-(71), for Eq. (67) at Q_0 = constant, one obtains respectively:

$$E \approx hp^{2/3} \approx 1/\alpha_T \tag{72}$$

$$E \approx h(\bar{T} + C)^{1/3}/[\bar{T}^{1/6}(\bar{T} + C_1)] \sim \bar{T}^{2.5}/[\alpha_T(\bar{T} + C)^{2/3}(\bar{T} + C_1)] \tag{73}$$

$$E \approx h\Delta T^{4/3} \approx \text{constant} \tag{74}$$

from which it immediately follows that at Q_0 = constant and $\bar{\delta}$ = constant: E is independent of ΔT; E decreases when $\bar{T}$ is lowered, although only slightly (this is obviously true only up to a maximum of $Q_0(\bar{T})$); and E decreases when there is an increase of the pressure for values higher than the maximum of $Q_0(p)$.

In order to estimate the dependence of E on $\bar{\delta}$, the values of δ and h should be taken from Eqs. (39) and (40) and substituted into Eq. (67). As a result by using Eq. (41):

$$E = (5/7)^{1/2}(\delta^3 + 2/\delta^3)\lambda B\Delta T \ln Q_0k_p/2\bar{\varepsilon} \tag{75}$$

Denoting $E = E_0$ at $\bar{\delta} = 1.0$ and assuming $k_p = 1.0$, one arrives at (in reduced units and constant p, T_H, T_C, and Q_0)

$$E/E_0 = 1/3(\bar{\delta}^3 + 2/\bar{\delta}^3) \tag{76}$$

It is not difficult to show that Eq. (76) has a minimum at $\bar{\delta} = 2^{1/6} = 1.122$, and in this case $E/E_0 = 0.943$, and for $\bar{\delta} = 1.0$ and $\bar{\delta} = 1.26$, $E/E_0 = 1.0$ (see Fig. 9). From these results and Eqs. (38) and (39), it is found that from an

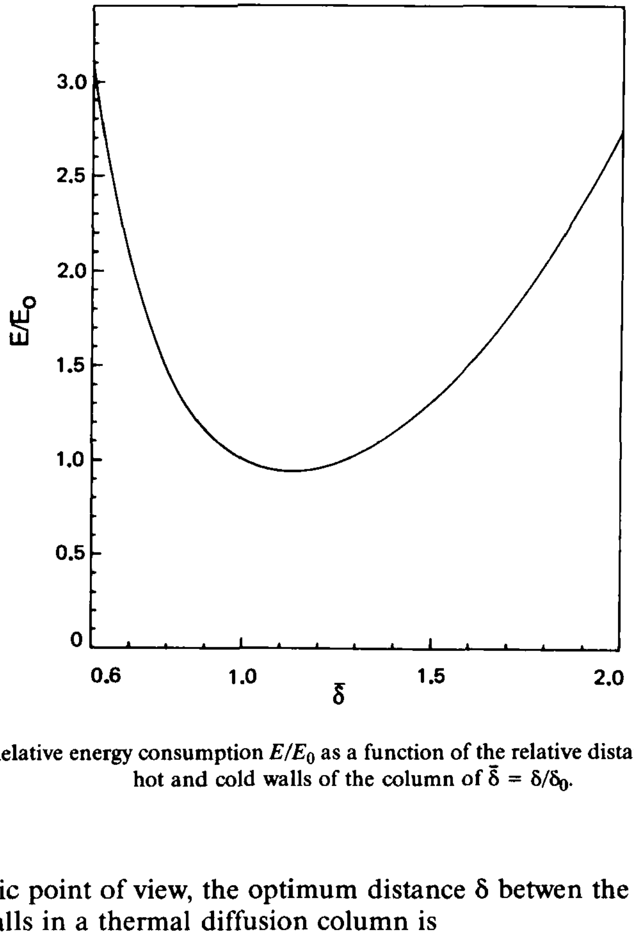


FIG. 9. Relative energy consumption E/E_0 as a function of the relative distance between the hot and cold walls of the column of $\bar{\delta} = \delta/\delta_0$.

energetic point of view, the optimum distance δ betwen the hot and the cold walls in a thermal diffusion column is

$$\delta_{\mathrm{opt}} = 2^{1/6}\delta_0 = [(9!)^{1/2}\eta D/(\rho g\beta\Delta T)]^{1/3} \qquad (77)$$

4. CONCLUSIONS

The analyses of the influence of the thermodynamic parameters of the gas mixture and the geometry of the column permits the optimization of the process of separation in a thermal diffusion column in practice. The results, based on general considerations about the dependence of the degree of separation on the principle parameters of the processes in a column, are checked and verified in the present work for the case of the separation of a He–N_2 gas mixture in a thermal diffusion column with cylindrical geometry. In principle, the generalization for an arbitrary gas mixture can be done from general considerations. The main conclusions are:

1. The dependence of Q_0 on pressure at $\bar{\delta}$ = constant has a character similar to that of $\alpha_T(p)$ (see Figs. 4–6) with a well-defined high maximum which is at a higher pressure than the maximum in α_T. This is natural and follows from Eq. (59). For a He–N_2 mixture the optimum pressures are between 1.0 and 1.5 MPa. Higher pressures would be necessary in columns with $\delta < 1$ mm, which would create serious problems in their realization.
2. From a technological point of view, the best value for T_C is in the interval 280 to 300 K, the optimum value for T_H depends on the particular gas mixture, and in the case of a He–N_2 mixture is in the interval 600 to 1000 K; higher values of T_H correspond to higher pressures. It was shown analytically and experimentally that the functions $Q_0(T_H)$ at T_C = constant and $Q_0(\bar{T})$ at ΔT = constant have a maximum.

These conclusions about the optimum conditions for gas separation in a thermal diffusion column for static conditions are valid for gas production in a thermal diffusion column, too. As shown in Ref. *6*, for the degree of separation in a thermal diffusion column in the dynamical regime, Q is proportional to Q_0, and to a first approximation:

$$Q - 1 \approx Q_0 - 1$$

This question will be discussed by the authors elsewhere.

SYMBOLS

B	geometrical width (average perimeter) of column (Fig. 1)
C, C_1	characteristic constants for a particular substance
D	diffusion coefficient in the separated gas mixture
D_0	diffusion coefficient at p = 100 kPa and T = 273.16 K
E	energy

F	cross section through which the heat flows
G	convection flow going down
g	earth's acceleration
H, K	constants introduced by Furry, Jones, and Onsager
h	geometrical height of the column
h_0	transfer unit height (TUH)
h_c	principle value of TUH
h_d	correction for longitudinal diffusion in TUH
h_p	correction for transverse diffusion in TUH
L	convection flow going up
M_L	molecular weight of the light component in a gas mixture
M_H	molecular weight of the heavy component in a gas mixture
N	number of TUH on the total height of the column
p	pressure
p_0	100 kPa
Q	degree of separation with gas production of column
Q_0	degree of separation without gas production
Q_H	degree of enrichment of heavy gas in the mixture
Q_L	degree of enrichment of light gas in the mixture
q	coefficient of separation
R	universal gas constant
r_1	radius of outer wall of cylindrical column
r_2	radius of inner wall of cylindrical column
$\bar{T}$	logarithmic averaged temperature in thermal diffusion column
T_H	temperature of hot wall in thermal diffusion column
T_C	temperature of cold wall in thermal diffusion column
T_{Hm}	average temperature of convection flow going up
T_{Cm}	average temperature of convection flow going down
ΔT	temperature difference between the hot and cold walls
$T_0, \bar{T}_0$	temperature of water triple point (273.16 K)
x_0	concentration of the light component in initial gas mixture
x_{L1}	concentration of the light component in lighter gas enriched fraction
x_{L2}	concentration of the light component in heavier gas enriched fraction
x_{H1}	concentration of the heavy component in lighter gas enriched fraction
x_{H2}	concentration of the heavy component in heavier gas enriched fraction

Greek Letters

α_T	thermal diffusion factor
β	coefficent of volume expansion
δ	distance between the hot and cold walls in a column
δ_0	distance between the hot and cold walls in a column for which Q = max for a particular mixture
$\bar{\delta}$	ratio of real distance δ to optimal δ_0
η	coefficient of kinematic viscosity
η_0	coefficient of kinematic viscosity at $p = 100$ kPa and $T = 273.16$ K
λ	thermal conductivity of a gas mixture
ρ	density of gas mixtures at $\bar{T}$ and the pressure at which the process is conducted
ε	coefficient of enrichment

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